Implementing a Low-Cost Sawyer-Tower Circuit to Measure Ferroelectric Properties at Various Frequencies

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Abstract

Design specifications of a low-cost Sawyer-Tower circuit with variable frequency are discussed. In addition, a comprehensive description of the fundamentals of how and why this circuit works is given, beginning with an introductory discussion on the basic physical principles concerning ferroelectricity. Methods for deriving important values from the polarization vs. electric field plot (P-E loop) are provided and results from testing a ferroelectric sample (PZT-5H) with a low-frequency Sawyer-Tower circuit setup are shown. The results that are obtained reflect the simplicity, feasibility, and accuracy of the Sawyer-Tower circuit.

Introduction

Historically, one of the first methods used to measure properties of ferroelectric materials was the Sawyer-Tower circuit, designed by C.B. Sawyer and C.H. Tower in 1930, only 10 years after ferroelectricity was first shown in Rochelle salt by Joseph Valesek and Pierre Curie [1]. The original design of the circuit consisted of a ballistic galvanometer, a voltage divider, a reference capacitor, and a DC voltage source [2]. With the advent of modern electronics, the original Sawyer-Tower circuit has undergone slight changes, but it possesses many of the same features as it did nearly 90 years ago. Furthermore, digital recording devices has made measuring polarization properties much easier and is commonly done for research and education purposes due to its simplicity.

In this essay, we will discuss a low-cost way to implement a Sawyer-Tower circuit and expand on its design. The fundamental physics behind how this circuit works, as well as a description of how one would extrapolate important values from the hysteresis loop (such as remnant polarization, saturation polarization, etc.) will be discussed. Additionally, ferroelectric samples will be tested in this circuit and results (in the form of fully-developed hysteresis loops) will be shown for various frequencies. But first, it is important to have a good physical understanding of the properties of ferroelectrics and how they are measured, the general layout of the Sawyer-Tower circuit, and other circuits that measure polarization before diving into the details of our circuit design.

Background

Ferroelectric Structure

Ferroelectric and pyroelectric ceramics are identified by their reversible spontaneous polarization [3]. This means that with an external field absent, a ferroelectric or pyroelectric substance is capable of possessing some finite internal polarization, which is made possible by a transformation in the lattice structure of the material. In fact, what differentiates a pyroelectric from a ferroelectric is the ease with which the lattice can be modified. The slight difference in crystalline structure allows ferroelectric materials to undergo transformations easily while pyroelectric materials often have energy barriers that must be overcome in order to alter the structure [3]. More specifically, the ease at which ferroelectrics can change structure allows them to be characterized by a second order phase transitions, unlike pyroelectrics which are a first order phase transition. This difference allows ferroelectric materials to possess special properties different from that of other perovskite ceramics [3]. While piezoelectrics, pyroelectrics, and multiferroics can be tested with Sawyer-Tower type setups, the discussion and experimentation will be strictly limited to ferroelectric materials.

Looking at the structure of a ferroelectric in more detail, one can see that it is of the perovskite structural form ABX_3, which can be seen in *Figure 1* [4]. The "A" represents smaller cations located in the corner of the unit cell, the "B" also represents a cation but of much greater size than A, and is located in the center of the unit cell. The "X_3" represents anions located on the face centered positions. In most cases, this is oxygen. As seen in *Figure 1*, the left image is a normal perovskite structure that is non-polarized. When an external electric field acts on a ferroelectric material, the positively-charged B cation in the center of the unit cell is displaced in a direction along the electric field. This forms the "ferroelectric structure", shown in the middle image. When many lattices are in this type of orientation, there will necessarily be some finite, measurable polarization in the bulk of the material [4]. The amount of this polarization will depend on the magnitude and direction of the externally applied field, the specific type of perovskite material, and the frequency of the field (if it is alternating) [5]. The goal of the Sawyer-Tower is to measure this; specifically, to plot the polarization against the applied electric field.

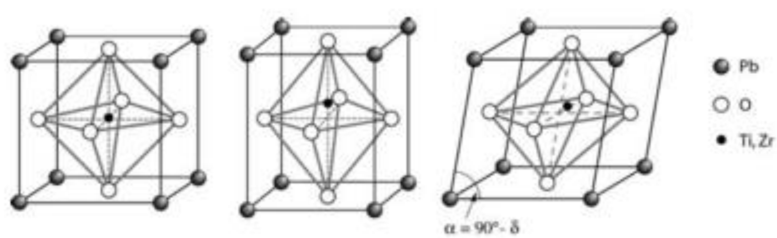


Figure 1: The perovskite structure (left), ferroelectric structure (middle), and rhombohedral structure (right) are common configurations of perovskite ceramics [4].

Polarization-Electric Field Plots

If one plots the polarization vs. electric field (P-E plots) for ferroelectric materials, one will see the well-known hysteresis loop, identical to H-M curves for ferromagnetics. This hysteresis loop contains valuable information about that specific material, such as remnant polarization, coercive electric field, and saturation polarization. The hysteresis loop acts like a fingerprint for each ferroelectric sample, and is thus significant to obtain in order to fully characterize a material [5]. Granted, there may exist factors that may change the ferroelectric properties of that material, such as grain size, sample fatigue, sample thickness, or any number of other variables. Despite this, the overall shape of the hysteresis loop will be fairly consistent [5].

It is often helpful to look at P-E loops for simple, linear circuit elements in order to better understand its significance. An ideal linear capacitor, ideal resistor, lossy capacitor, and non-linear ferroelectric are shown in *Figure 2* below [6].

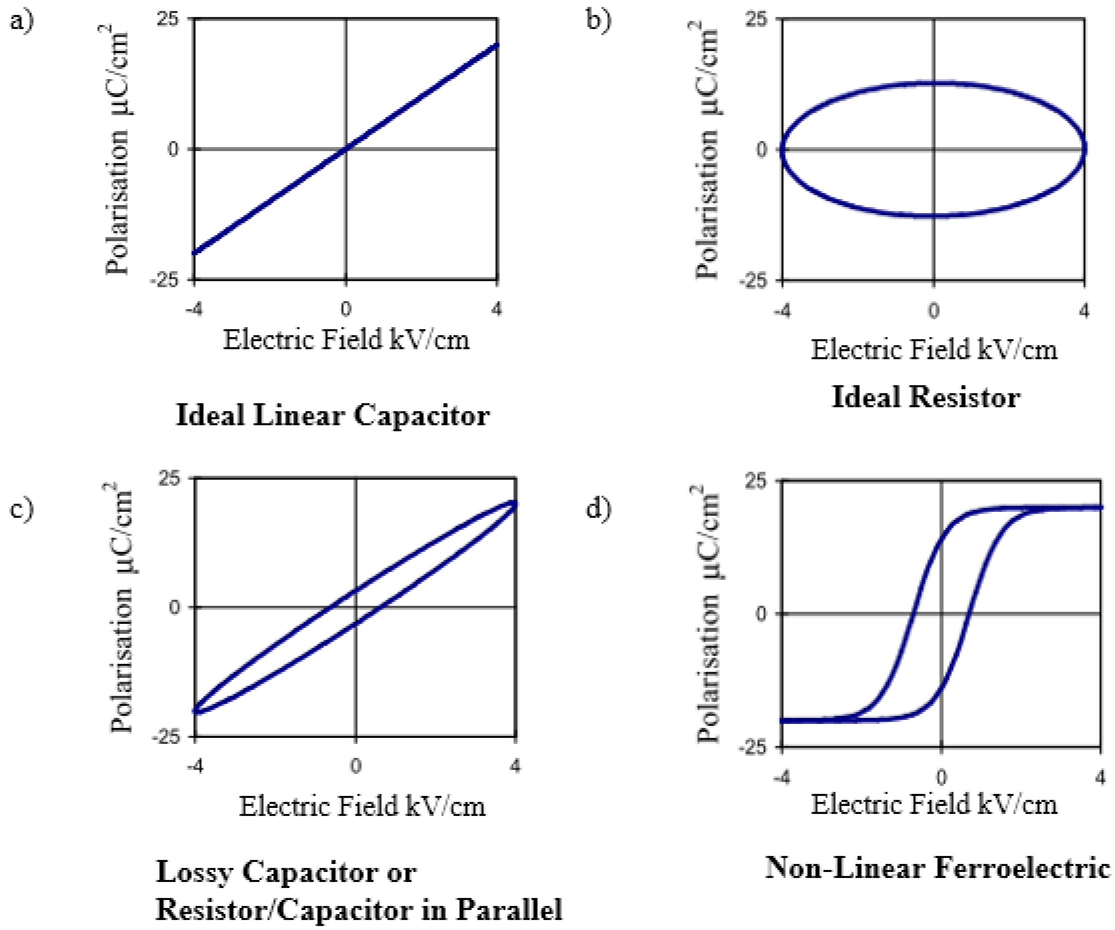


Figure 2: P-E Hysteresis Loops for various simple circuit elements [6].

An ideal capacitor (*Figure 2a*) with a dielectric substrate between plates will show a perfectly linear response, as one would expect with a dielectric material. As the electric field increases across the plates, so does the displacement field, and thus the polarization. Also, for an ideal capacitor, the current leads the voltage by 90 degrees, thus making the charge in phase with the voltage since charge is the time integral of current [6]. Note that the charge developed across the plates is intimately related to the displacement field and polarization. In fact, some studies will plot charge against electric field and achieve the same hysteresis shape [7]. For an ideal resistor (*Figure 2b*), the current and voltage are perfectly in phase, making the charge 90 degrees out of phase. The P-E plot for this will appear as a circle with the center at the origin. For a lossy capacitor, or for a circuit containing both resistive and capacitive elements in parallel, one would see a skewed ellipse (*Figure 2c*). This plot is actually very important to the Sawyer-Tower circuit in that it can compensate for stray capacitances and conductive losses in real ferroelectric samples when added in parallel, but this will be discussed in more detail later [8]. Finally, the non-linear ferroelectric capacitor hysteresis loop is shown above (*Figure 2d*) and demonstrates that polarization can exist even if the applied field across the ferroelectric capacitor is zero [6]. This fits the definition of ferroelectrics, as they are identified by their reversible spontaneous polarization. It is important to note that for capacitors, the capacitance is directly related to the *slope* of the P-E loop [4], [6]. Note that for the linear capacitor, the slope (or the derivative of polarization with respect to the electric field) is constant. This constant capacitance is exactly what one would expect for a capacitor with a constant relative permittivity. The ferroelectric capacitor demonstrates interesting properties. Specifically, that the relative permittivity is *not* constant throughout the hysteresis loop and thus the relative permittivity and electric susceptibility will be functions of electric field [9].

Before going any further, it is imperative to begin discussing the important values that can be extracted from a hysteresis loop. Shown below in *Figure 3* is a generic hysteresis that one would expect to see with a typical ferroelectric sample [6].

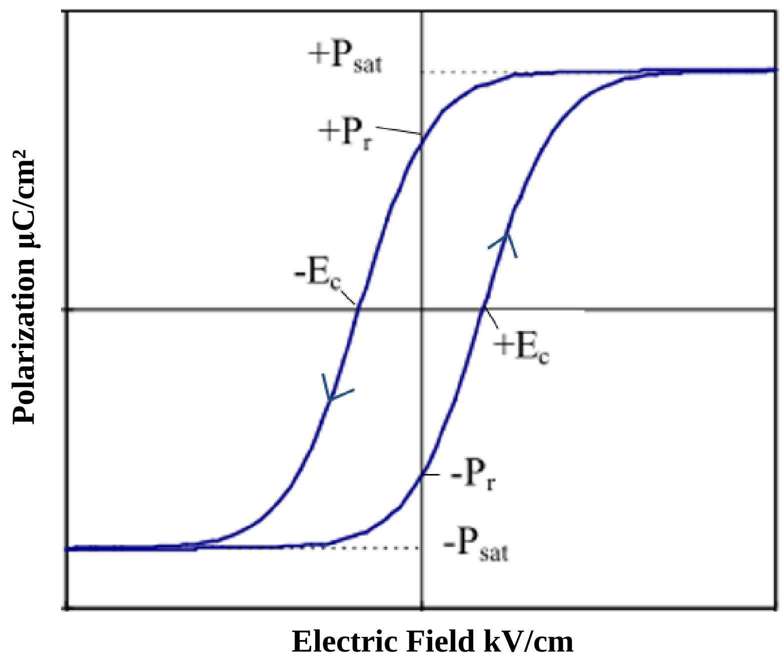


Figure 3: Generic hysteresis loop with important hysteresis values shown. These values serve as reference points to help characterize various ferroelectrics [6].

As shown above, the hysteresis loop will repeatedly progress in the direction of the arrows as time progresses if the electric field is periodic, which is true for almost all cases. The important values to look at on a ferroelectric hysteresis plot are the coercive electric field strengths, saturation polarization, and remnant polarization. The coercive field strengths (-Ec and +Ec as shown in *Figure* 3) are the values associated with the hysteresis loop crossing the E-axis. Physically, this means that when a ferroelectric material is already fully polarized in one direction, then the coercive field strength is the magnitude of the electric field with which the material must become non-polarized. This is drastically different from that of a linear dielectric capacitor, in that when there is no electric field, then there is no polarization. With that said, the remnant polarization (-Pr and +Pr as shown in *Figure* 3) refers to the amount of finite polarization that persists in the material when the electric fields drops to zero. This is the point where the hysteresis loop crosses the y-axis. In fact, this is one of the most important characteristics that distinguishes ferroelectric materials from non-ferroelectric materials [3]. Lastly, the saturation polarization (Psat) refers to the maximum polarization that a material can obtain. If a material is at a point of saturation polarization, then increasing the magnitude of the electric field will have no effect on further increasing polarization [10], [11]. This value is often misinterpreted as the maximum (or minimum) polarization value on a hysteresis loop [2], [5], [6]. This is not always the case, especially if the material has losses, as most practical ferroelectric capacitors do [2]. When the material has losses, the slope of the hysteresis loop at the polarization and electric field extremes is not exactly horizontal, but tilted. The *true* saturation polarization of the material is the value at which the slope of the hysteresis loop (when the material is beginning to be repolarized) intersects the y-axis. In *Figure 3,* the ferroelectric is ideal and the dotted line, which represents the slope at the extrema, is exactly parallel and intersects the y-axis when the polarization is at its minimum. To reemphasize, this dotted line will not always be exactly parallel in realistic ferroelectric capacitors unless a compensation component is added to account for conductive losses and stray capacitances, which many Sawyer-Tower circuits include [6].

Sawyer-Tower Circuit Schematics and Measurement

Now, let's begin looking at the basic schematics of the basic Sawyer-Tower circuit. As was mentioned previously, the circuit itself has not changed drastically since its inception, aside from the inclusion of compensation circuits (which account for sample losses) and digital measuring devices. *Figure 4* is an example of a basic Sawyer-Tower circuit [6]. This is, by far, the most commonly used apparatus to measure polarization in ferroelectrics due to its simplicity and low-cost [2].

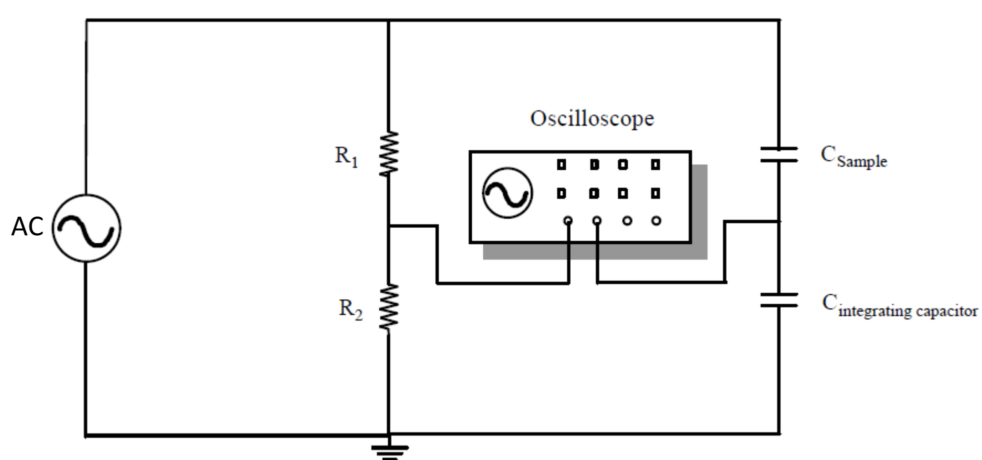


Figure 4: Basic schematic of the Sawyer-Tower circuit. Its simplistic design allows for easy replication of studies.

As shown in the circuit above, test probes are measuring voltage between the two resistors and the two capacitors. The voltage information that is obtained between the resistors is sent to the x-channel of the digital oscilloscope and the voltage information that is obtained between the two capacitors is sent to the y-channel. These two channels are plotted against each other (in x-y mode) and, if the voltage scaling, trigger functions, and time divisions are set correctly on the oscilloscope, a hysteresis loop will appear in real time. One must realize that the loop that is being seen on the oscilloscope is actually a collection of individual test points that are constantly being acquired and is thus not a continuous line, although it appears to be since the data points are acquired at such a fast rate. It is important realize that the hysteresis loop is completely time dependent and the data points naturally progress periodically along the hysteresis loop, as was demonstrated with the arrows in *Figure* 3.

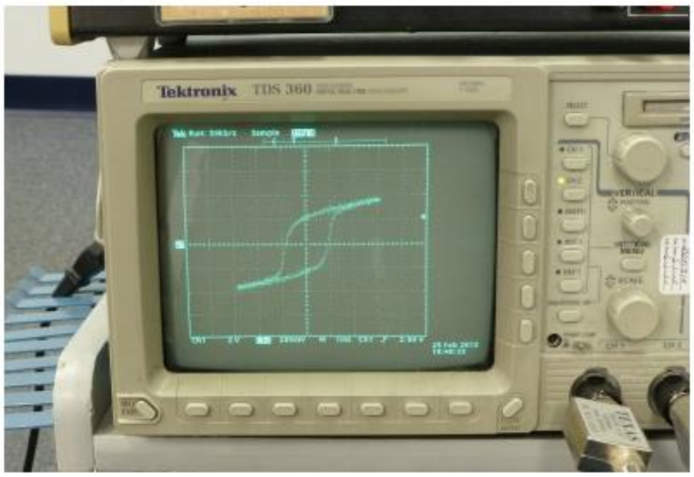


Figure 5: A real-time hysteresis loop displayed on a digital oscilloscope [7], [12].

Looking at the circuit components more closely, there are a set of two resistors in series and a set of two capacitors in series, which are in parallel with one another. One of the primary functions of the two resistors in series (the voltage divider) is to serve as bleeder resistors for each capacitor [2]. This is more-or-less a safety function that allows the capacitors (which can have relatively high capacitances) to discharge quickly into the resistors, instead of holding charge and posing an electrical hazard. In fact, one can effectively exclude the resistors from the circuit and still achieve polarization measurements since they are not functional to the P-E loop measurements [2]. These resistors also serve to attenuate some of the high voltage signal, allowing one to protect the probes that carry the signal to the oscilloscope [2]. The two capacitors consist of the ferroelectric sample, which is placed in a capacitive geometry by depositing a conductive substrate onto both sides of the ferroelectric material, and a reference capacitor. The reference capacitor, which necessarily must have a much higher capacitance than the sample capacitance, serves to integrate the current into charge [6]. Thus, the voltage measured between the capacitors is directly proportional to charge. Likewise, the voltage drop across the sample capacitor is directly proportional to electric field between the capacitor. This direct proportionality is the primary reason why one can visualize the hysteresis loop on the oscilloscope in real time, even though the parameters being displayed are voltages and not true

P-E plots. A more thorough discussion on this will come in the methods section, where we will provide a mathematical treatment for voltages, polarization, and electric fields.

Other Circuits to Measure Polarization

One of the benefits of the Sawyer-Tower circuit is that the large reference capacitor readily integrates current into charge, and thus produces an entire hysteresis loop [6]. This is opposed to other methods, which require current to voltage converters or software that integrates current into charge [13]. While other methods may require more equipment and are often more tedious to perform, they can offer distinct advantages.

For instance, one circuit that is fairly common was proposed by Dickens et al. which utilizes a unipolar and bipolar field in conjunction [13]. When a ferroelectric material is switched from a positive polarization to a negative polarization, the physical flipping of the dipoles in the material actually induce a polarization *current*. Since current is defined as the movement of charge, this polarization current physically arises from the B cations (as was discussed earlier) moving spatially with the electric field. This current can be measured by subtracting the unipolar field from the bipolar field. The unipolar field switching only contains information about the capacitive response of the material, and not the ferroelectric dipole switching but the bipolar field *does* contain ferroelectric switching information when the field is reversed. The current that is measured can be processed using a current to voltage converter, and the hysteresis loop values (remnant polarization, coercive electric field, etc.) can then be calculated by a computer algorithm. This technique is beneficial in that it does not require as much equipment as the Sawyer-Tower method and the data can be achieved in a very efficient manner [6]. The downsides to this method is that it does not produce the full hysteresis loop, but only hysteresis values, and it requires a computer algorithm to process the information.

A very similar method has been proposed by Dias and Das Gupta, but instead of digitally subtracting unipolar from bipolar fields, it utilizes built in circuitry to do so [14]. This method requires three identical ferroelectric samples to be placed in parallel. Specifically, a bipolar field, a positive unipolar field and a negative unipolar field will be applied across each of the three samples. Hardware implementations are added to subtract the capacitive responses from two of the samples and only allow only the polarization switching from the third sample to be readout. This method is designed to be a more convenient way to measure the polarization of a ferroelectric material with non-ideal capacitive or conductive behavior, specifically materials with relatively low polarization and high permittivity and/or conductivity [14].

There also exists way to account for non-ideal capacitive and conductive behavior for the Sawyer-Tower circuit with an addition of the aforementioned compensation circuit [2], [8], [9]. This is called a modified Sawyer-Tower circuit and requires an additional resistor and capacitor that are placed in parallel with the entire circuit. This compensation circuit has leads running from the signal input to where the voltage is read between the capacitors. This, in effect, generates reverse phase of currents which compensate for losses by subtracting them out at the voltage readout location [2]. This can be visually seen below in *Figure 6*, where the P-E loop of the resistor, capacitor, or lossy capacitor (which is equivalently a resistor and capacitor in series) can compensate for the slight deformations in the true P-E loop acquired from the lossy ferroelectric sample [2].

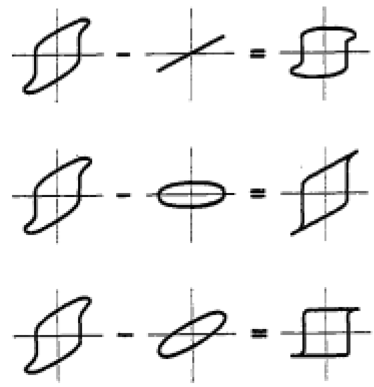


Figure 6: Deformed hysteresis loop with an addition of reverse phase currents to compensate for ferroelectric losses. The bottom right image shows an ideal hysteresis curve [2].

There are also circuits in which virtual grounds and transimpedance amplifiers can be used to probe for transient polarization currents [11], [15], [16]. These circuits are what are mainly used in commercial systems as they can allow for exact measurements, and even more exact when used in conjuction with highly developed software [6]. There exists two major companies which sell ferroelectric testers: Radiant Technologies and aixACCT [5]. These systems also have their limitations in terms of maximum voltage that the system can apply, not to mention these systems can become costly, especially when a high voltage interface and software are included. Although these systems are subject to higher cost, it is common for studies to test their circuit's results with an accurate commercial system, lending liability to their own system's performance.

When using any of these circuits to measure polarization, one prominent challenge is to safely implement a high voltage amplifier that provides sufficiently high voltage (and hence electric field) across the sample [6]. This is mainly the case for thick samples, which require higher voltages to achieve the same field strengths as thinner materials. For samples of any size, electric fields across the sample will often need to reach close to 100 kV/cm to achieve full saturation polarization [5]. For instance, a sample that is 200 microns thick will require 1000 V to achieve a reasonable field strength of 50 kv/cm. This is done by the simple relation that $E = V/d$ for parallel plate capacitors, where d is sample thickness, V is required voltage across sample, and E is electric field strength. Many setups will immerse their samples in oil to reduce arcing of the sample if voltages are relatively high [2], [8], [14]. Another important consideration when using high voltage is that high voltage probes must be used in order to safely extract signal information from the circuit without damaging the oscilloscope. To this end, the other components (resistors, capacitors, and wires) in the circuit must also be rated for high voltage, otherwise risking damage during testing. In any case, it is most important to calculate the necessary voltages that will be required before testing.

Methods

Low Frequency Setup

The setup that was used for this experiment was the basic Sawyer-Tower method, in which two 10 kΩ resistors were placed in series. Keep in mind that it is possible to use different values of resistance. These resistors were wired in parallel to two capacitors in series, one of which is the sample while the other is a large capacitor (1 μF). This reference capacitor is relatively large compared to the sample capacitance, which was measured to be ~5 nF with a digital LCR multimeter. A picture of the circuit helps to illustrate the situation. Looking at *Figure 7*, one can easily identify the two resistors and two capacitors mounted on a perfboard. This board utilizes terminal block connectors, which are electrically connected underneath the board. Using these connectors allow the user to easily interchange the components of the circuit, which is convenient if samples are continually being tested in the circuit.

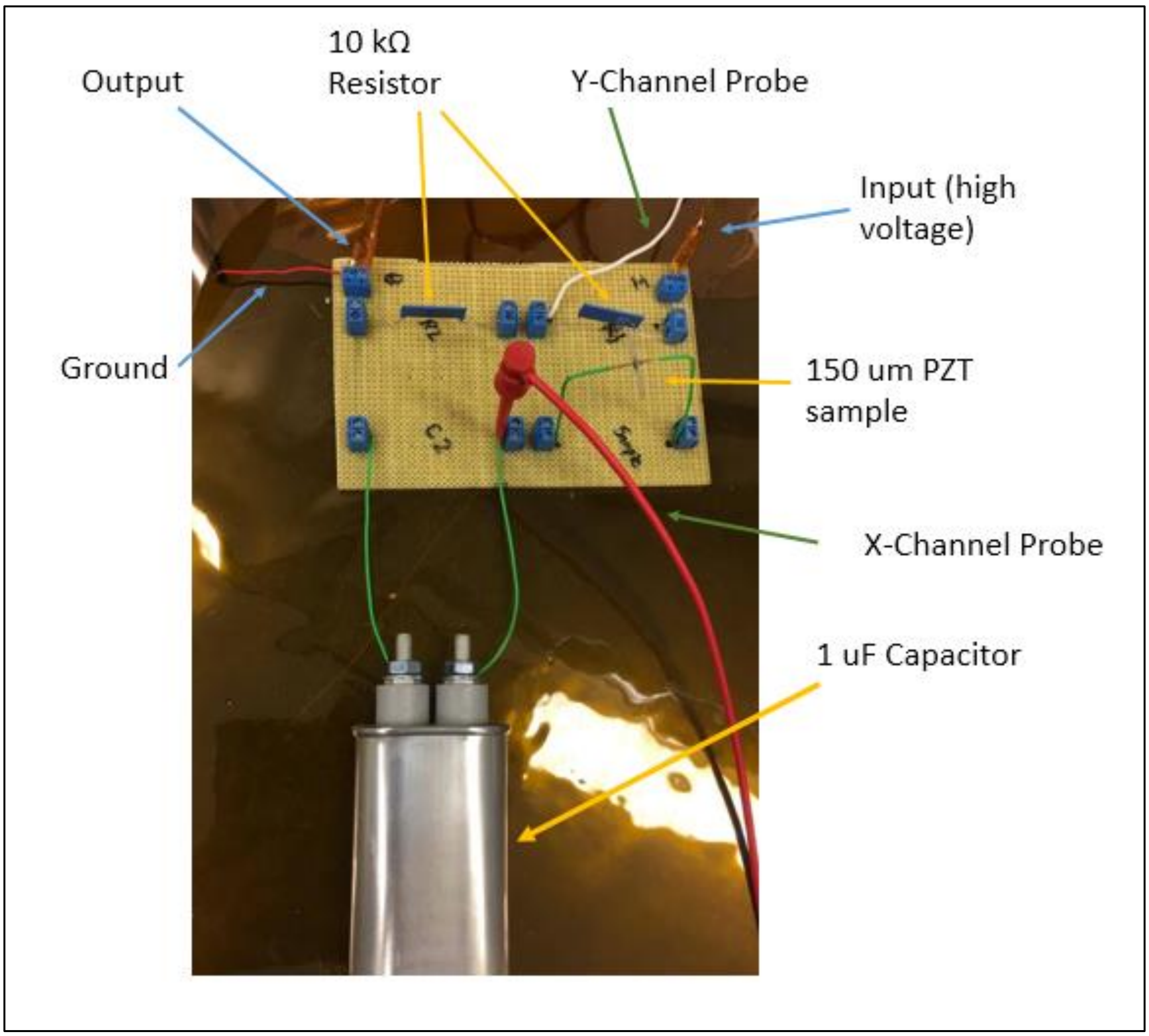
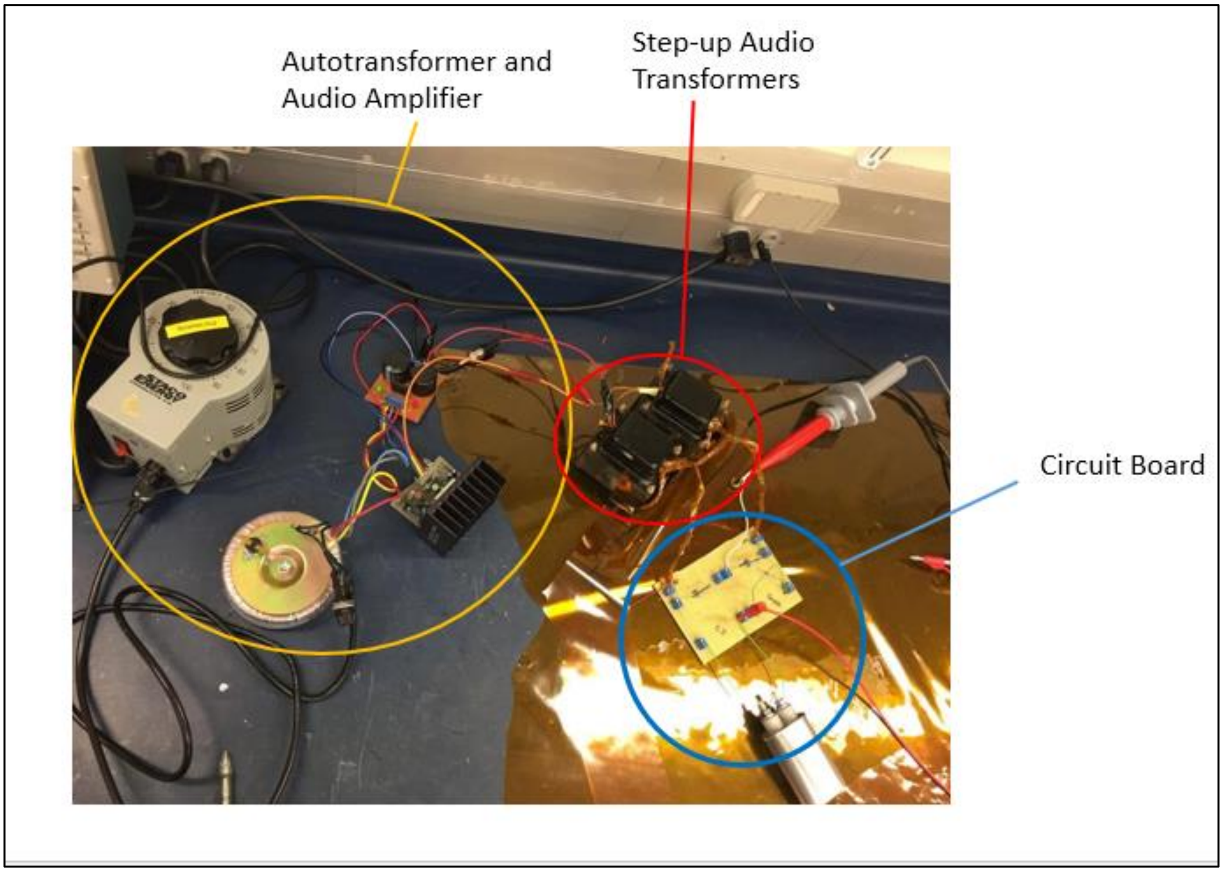


Figure 7: Sawyer-Tower Circuit layout.

The circuit was placed on a sheet of Kapton, and the input/output transformer wires were also wrapped in Kapton tape to prevent arcing. Since the sample was relatively thick, transformers and an audio power amplifier were used to increase the voltage from the signal generator which had a maximum 10 V output, to the circuit which was capable of outputting ~2000 V after amplification.



The audio amplifier and steel core transformers shown above were salvaged from an unused audio system. The two transformers (each with ~100x gain) were wired in parallel to produce a total of ~200x gain. The autotransformer (Staco Energy Model 3PN1010B) was used to regulate power to the audio amplifier, controlling the signal that was sent to the circuit. The signal that was read in the circuit from the high voltage probes was then sent to a digital oscilloscope which could read out the hysteresis in real time.

The ferroelectric material that was used for testing was $Pb(ZrTi)O_3$, namely PZT-5H. The material came in strips, each 150 microns in thickness. These strips were cut into reasonable small squares, about 8mm in width and 5 mm in length. Silver was etched on the back of these PZT samples and leads were attached to the conducting silver, creating a parallel plate capacitor in effect. The PZT sample must be refrained from reaching its Curie temperature (195℃) so as to not burn the sample and destroy the ferroelectric properties of the material. These temperatures can be easily reached with a low power soldering iron and hot solder. Thus special precautions must be taken so that the leads are safely soldered on without compromising the ferroelectric properties. The simplest way to attach leads to the back of these small samples was to wrap the edge of the sample in Kapton tape, leaving a smaller exposed square where the solder can be placed. This is so the solder will not run to the edge of the sample, which can frequently happen if Kapton tape is not placed at the edges. Place a small amount of rosin flux on the exposed area of the sample as well as one the wire that is to be attached to the sample. Remove a section of plastic (PVC) from the wire and form a small bead of flux at the end of the wire using a soldering iron. Tinned copper wire was used in this experiment, since any problematic high frequency skin effect will not be a factor in this initial experiment. Heat the copper wire with the soldering iron. Once the copper wires is sufficiently hot, remove the soldering iron from the wire. Place solder on the iron and place it on the wire, so a bead of solder is formed. Immediately place the hot bead of solder onto the sample, which should form an electric connection. This same procedure can be done for the other side of the sample as well. Be careful when making contact with the sample, or when moving the sample, due to its fragile nature. Once both sides have been successfully soldered (most likely after a few attempts), the Kapton tape can be removed from the edges. Due to the thinness of the samples, an insulating liquid (EPO-TEK OE132-43 Epoxy) was used to prevent high voltage arcing. Many other studies have done similar procedures, most notably, submerging the sample in mineral oil. Additionally, other studies using relatively thick samples have successfully used spring-loaded holders to ease the process of exchanging samples [2], [8].

One may wonder about the importance of having a “relatively large” capacitor in series with the sample, and how exactly that capacitor integrates the current into charge. This can be easily explained by exploiting Kirchhoff’s Laws. Note that although Kirchhoff’s most specifically applies to DC circuits, at low frequencies such as the ones that are being used in this initial experiment, the AC case is very well approximated by Kirchhoff’s Laws.

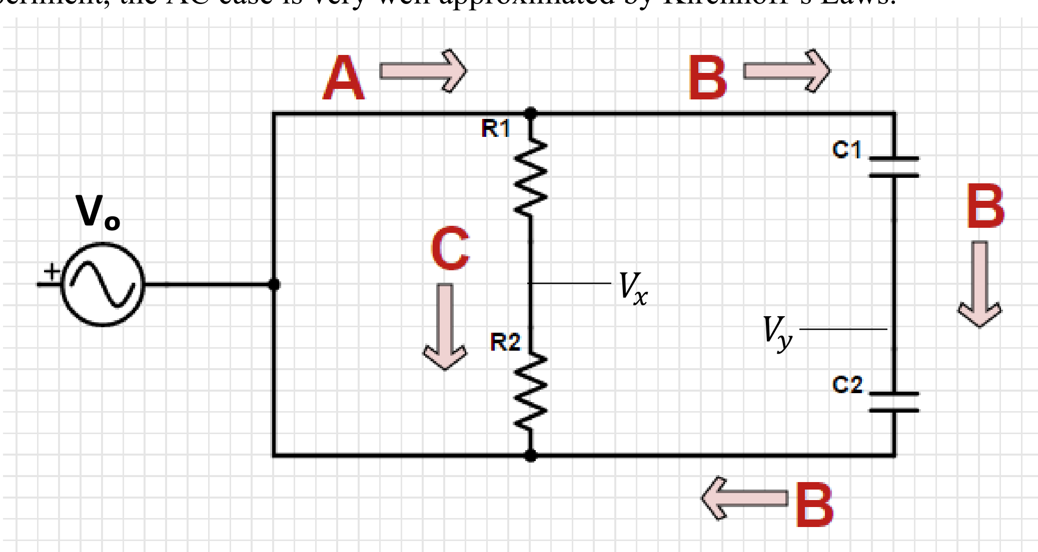


Figure 8: Schematic of Sawyer-Tower Circuit, courtesy of DigiKey. Utilizing Kirchhoff’s Law, one can deduce the current in each branch of the circuit.

Looking at *Figure* 8, R1 and R2 represent the resistors, C_1 represents the ferroelectric capacitor, and C_2 represents the reference capacitor. One should choose a reference capacitor that has a much greater capacitance than the sample. Sample capacitance can be readily measured with a digital LCR multimeter. Although it is expected that the capacitance of the sample will change as applied electric field changes, a measurement with a digital multimeter should provide a good frame of reference as to how large the reference capacitor needs to be. Ideally, it should be 100 to 1000 times larger than the sample capacitance [6]. The reasoning behind this lies in Kirchhoff's Laws. Kirchhoff's Current Law tells us that the sum of all currents flowing into a node is equal to the sum of the currents flowing out of that node. This is simply conservation of electrical charge. In this case, current labelled A in *Figure 8* should be equal to current B plus current C. Thus, the branch of the circuit that contains the capacitors must have constant current B. Now, one can use Kirchhoff's Voltage Law to give us the relative voltage drops across the capacitors in series. Kirchhoff's Loop law is a derivative of conservation of energy and tells us that the algebraic sum of the voltages in a loop around a circuit (from one point, around any loop in the circuit, back to that same point) must equal zero. When capacitors are used in an AC circuit, it is best to use capacitive reactance to describe voltage drop using Ohm's Law:

$V = IX$ (Ohm's Law for Capacitors)

where V is the voltage, I is the current, and X is the capacitive reactance. In this case, the only concern will be the far branch of the circuit in *Figure 8* and thus, the only current that will be of concern is current B. Assume that the circuit will have some known input voltage that will be called V_o. Now, all that is needed is the capacitive reactance (X) for each capacitor. I will call the reactance of the sample capacitor X_1 and the reactance of the reference capacitor X_2. Reactance can be calculated in an AC circuit as shown below:

$$X = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

where ω is the angular frequency of the drive voltage, f is the signal frequency, and C is the known capacitance. As stated before, C_2 must be some magnitude larger the C_1. Thus:

$$C_2 = \alpha C_1 \text{ and } \frac{C_2}{C_1} = \alpha$$

$$X_1 = \frac{1}{2\pi f C_1}$$

$$X_2 = \frac{1}{2\pi f C_2}$$

$$\frac{X_1}{X_2} = \frac{C_2}{C_1} = \alpha$$

$$X_1 = \alpha X_2$$

With Kirchhoff's Voltage Law:

$$V_0 - V_1 - V_2 = V_0 - BX_1 - BX_2 = 0$$

where V_1 is the voltage drop across the sample, V_2 is the voltage drop across the reference capacitor, V_o is the known input voltage, and B is the current. Since $X_1 = \alpha X_2$, then:

$$V_0 - V_1 - V_2 = V_0 - BX_1 - BX_2 = V_0 - (B\alpha X_2) - BX_2$$

It is now obvious that $V_1 = B\alpha X_2$ and $V_2 = BX_2$. This leads us to the important relation that:

$$V_2 = \frac{V_1}{\alpha}$$

$$V_0 - V_1 - \left(\frac{V_1}{\alpha}\right) = 0$$

$$V_0 = \frac{V_1(1+\alpha)}{\alpha}$$

$$V_0\left(\frac{\alpha}{1+\alpha}\right) = V_1$$

This equation tells us that the voltage drop across the sample (V_1) will be approximately equal to the input voltage (V_o) if α is large. Looking back, to make α large, one must have a large reference capacitor. This is desirable because when one creates the P-E plot, the electric field that is plotted is directly related to the voltage drop across this capacitor. For instance, if the reference capacitor is 100 times greater than the sample capacitance (α = 100), then 99% of the input voltage goes into the voltage drop across the sample. Thus the calculated electric field is 99% accurate when using the relation $E = V_o/d$ where d is the sample thickness. To fully calculate E from the voltage measured in the x-channel of the oscilloscope, one must actually back out what the input voltage is. The voltage being measured between a voltage divider is <u>not</u> the voltage incident on the ferroelectric sample. If using measuring between a voltage divider, use the relation for a simple voltage divider (two resistors in series):

$$V_0 = (V_x) * \frac{(R_1 + R_2)}{R_2}$$

If $R_1 = R_2$, then the relation above simply leads to $V_o = 2V_x$. For the circuit in *Figure 8*, the equation to fully calculate the electric field across the sample becomes:

$$E = \frac{V_0}{d} = \frac{2(V_x)}{d}$$

A more generalized formula leads to the final result:

$$E = \frac{(V_x)}{d} * \frac{(R_1 + R_2)}{R_2}$$

Additionally, when an input voltage is applied across the ferroelectric capacitor, charge will build on the top plate of the sample capacitor. As the voltage alternates, the same amount of charge must leave the bottom plate of the sample and build on the top plate of the reference capacitor. This leads to the generalization that, in an AC circuit, the charge on the reference capacitor is the same as the charge on the ferroelectric sample at any time.

Now it is time to form a solid relationship between charge and polarization. For a parallel plate capacitor Q = CV. As stated previously, the charge on the sample capacitor and the reference capacitor should be equal. The voltage that is measured in the y-channel is the voltage incident on the reference capacitor. Thus, the charge on the reference capacitor ($Q_{reference}$) is calculated by $Q_{reference} = C_2 * V_y$. The charge on the sample (Q_{sample}) is equal to the charge on the reference capacitor. Thus:

$$Q_{reference} = Q_{sample} = C_2 * V_y$$

The displacement field for a parallel plate capacitor can be calculated using Gauss' Law:

$$Q_{free} = \oint \vec{D} \cdot d\vec{A} \quad \text{(Gauss Law for Displacement Field)}$$

where dA is an infinitesimal area vector that will be evaluated over the entire capacitor area and Q_{free} simply represents the charge on the sample. The area of the plates can be easily measured. This leads to the result:

$$D = \frac{Q_{sample}}{A}$$

It is also known that displacement field is related to polarization with this equation:

$$D = \epsilon_0 E + P$$

Since ε_o is on the order of 10^{-12} and E is at most on the order of 10^{8} (V/m), one can say that D is approximately equal to P.

$$D \simeq P$$

$$P \simeq \frac{Q_{sample}}{A} = \frac{C_2 * V_y}{A}$$

This shows that from known values, such as sample area, sample thickness, reference capacitance, and measured x- and y-voltages, one can mathematically transform a hysteresis loop displayed on the oscilloscope to a P-E loop that is useful for characterizing ferroelectric ceramics. Furthermore, it is easy to see that the P-E loop is directly proportional to the voltage-voltage loop displayed on the oscilloscope.

$$P = \frac{C_2 * V_y}{A}$$

$$E = \frac{(V_x)}{d} * \frac{(R_1 + R_2)}{R_2}$$

Lumping all of the constants together, it is easy to see the proportionality:

$P = \beta * V_y$

$E = \gamma * V_x$

where $\beta = \frac{C_2}{A}$ = constant and $\gamma = \frac{R_1+R_2}{d*R_2}$ = constant

In fact, this is the underlying reason that one can see a hysteresis loop on the oscilloscope. If there was no proportionality, the loop would look distorted on the oscilloscope until further calculation.

Higher Frequency Sawyer-Tower Circuits

The circuit described above is excellent for low frequencies, specifically, from about 1 Hz to 20 kHz. Above these frequencies, issues with the amplifier arise and the steel core transformer begins to not respond. An effort has been made to extend the frequency range of this circuit by simply swapping the steel core transformer with a ferrite core transformer, which can respond up to 250 kHz. In addition, another amplifier must be used since audio power amplifiers only operate up to ~20 kHz.

When going from the kHz range to MHz frequency range, issues can arise from stray inductances caused by high frequency, skin effect within wires, inductive heating in components, and differences in cable lengths range [15]. Another complication that may arise is relatively low circuit impedance, mostly depending on the capacitance of the sample since it has the lowest capacitance value. When operating at high frequencies, the circuit will demand a fairly high power requirement to achieve a suitable voltage across the sample. This, and the reasons stated above, are some of the reasons why there is a lack of research in the high frequency realm using the conventional Sawyer-Tower method [17]. Commercial testers are often limited in the frequency range they provide, typically only allowing up to hundreds of kHz. There are some studies which go into the MHz and GHz range, but these studies utilize a slightly different approach, such as using virtual ground circuits to probe fast polarization switching [15]–[17]. This study aims to find the frequency limits of the conventional Sawyer-Tower method by taking a strategic approach towards measurement and circuit design.

To test higher frequencies, two additional circuit designs have been devised so that they will overlap in frequency with the previous design (low frequency Sawyer-Tower) as well as overlap with each other to provide a continuous frequency range in which samples can be tested. *Figure 9* outlines the overlap of each Sawyer-Tower circuit.

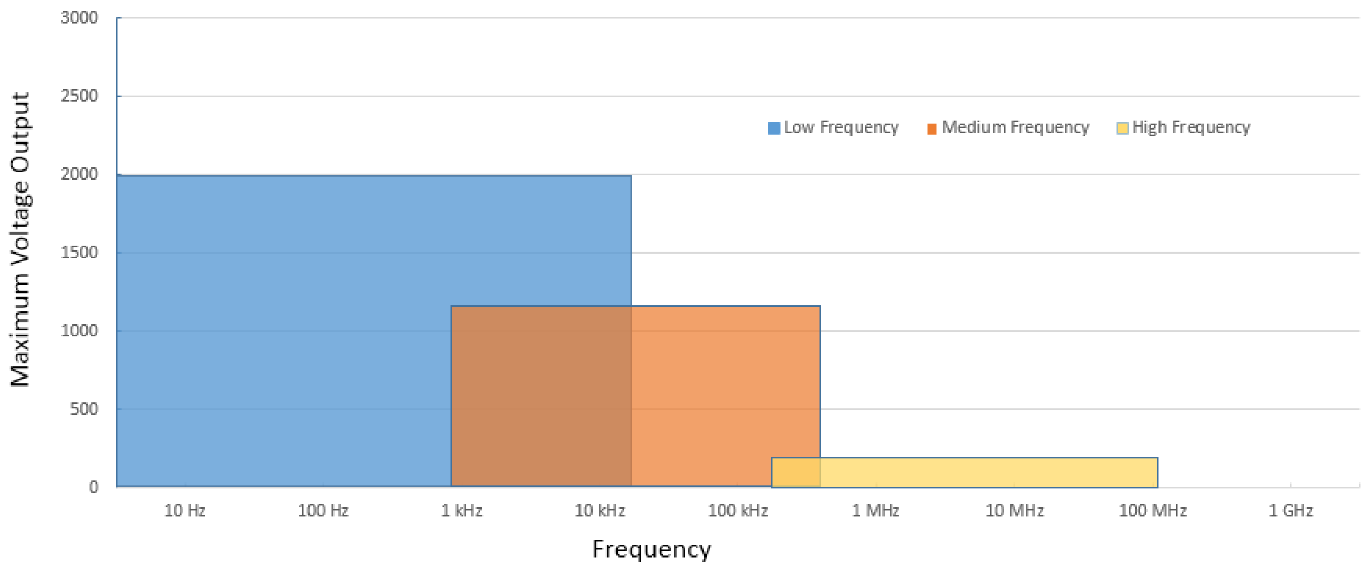


Figure 9: Operable frequency ranges with various Sawyer-tower systems.

There will, in total, be three fully operational systems that each span a specific frequency and each with a maximum output voltage. As the frequency range of the system increases, the voltage output of the system decreases. This requires using thinner samples to achieve a suitable electric field across the ferroelectric capacitor. In other words, if the thickness of the sample is decreased, the electric field across the increases due to the E = V/d relation. It is desirable for most P-E loops to have reached saturation, which is typically on the order of 50-100 kV/cm for most materials [6].

It is perhaps easiest to begin by discussing the medium frequency system. This system will be very similar to the low frequency Sawyer-Tower system that was described above, except that the audio amplifier will be replaced with a power operational amplifier (Apex PA09), capable of reaching 1 MHz signal output. As was mentioned previously, the steel core transformers will be replaced with ferrite core transformers. This will allow us to reach high voltage up until the transformers fail to respond, which should be close to 300 MHz. The operational amplifier can still output at higher frequencies, but this system will not be able to produce high voltage if the transformers are not responding, and will thus be ultimately restrained by frequency response of the ferrite core transformers. Moreover, components in the circuit will be replaced by non-inductive components and the copper wires will be replaced with braided Litz wire to reduce skin effect. It is important to have an oscilloscope and high voltage probes that are capable of achieving high frequencies. Again, this system will be very much the same as the low frequency system that was described above and the board layout will be identical. The medium frequency system will also be placed on a perfboard. But when nearing the MHz range, a different approach must be taken in order to achieve suitable measurements.

The schematics of the high frequency Sawyer-Tower system will be similar to the lower frequency systems, but will instead utilize a printed circuit board (PCB) rated for high frequencies. The components will consist of small surface mount capacitors and resistors, with the PCB board itself only spanning a few square inches. These components are rated for high power, since a 300 watt radio frequency power amplifier (ENI 3200L) will be used to increase voltage in the circuit. The reason that this high of power is required is due to the low impedance of the circuit when operating at such high frequencies. One can calculate the impedance (Z) in the circuit by forming equivalent capacitances, equivalent resistances, and taking the real part of this impedance (as impedance is naturally represented as a complex quantity). But before going into the calculation of impedance, first note that the capacitance of the sample can change with area and thickness, as is shown in this relation:

$$C = \frac{A\epsilon_r\epsilon_0}{d}$$

Since the capacitance of the sample is directly dependent on area and thickness (which is a variable that can be changed), the complex impedance must also be a function of these variables, as well as frequency. Since the equivalent resistance adds linearly, the equivalent reactance (from the capacitors) adds as reciprocals, and the two equivalent components are wired in parallel, one can show that the *real part* of the complex impedance is as follows:

$$Z(f, A, d) = \frac{1}{\sqrt{\left(\frac{1}{R_{eq}}\right)^2 + \left(\frac{1}{X_{eq}(f, A, d)}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{R_{eq}}\right)^2 + \left(2\pi f C_{eq}(A, d)\right)^2}}$$

Recall that the relationship between electric field and voltage is E = V/d or Ed = V. If there is an electric field that is desired to be achieved across the sample, then the voltage required will be directly proportional to the thickness. Thus, voltage required will be a function of thickness d. Finally, one can use the power relationship:

$$P = \frac{V^2}{Z}$$

where P is power (in watts), V is voltage, and Z is impedance (in ohms). If the thickness and area to allowed to change, then the following formula results:

$$P(f, A, d) = \frac{V(d)^2}{Z(f, A, d)}$$

So, for a known power output, one will see that increasing the frequency will result in demanding more voltage across the sample to achieve an appropriate electric field due to the inverse relationship between frequency and impedance. Increasing the area of the sample and decreasing the thickness of the sample will also demand an increase in voltage to produce the desired electric field. This means that it is desirable to have thinner samples and smaller area, so that the output voltage is high enough when operating at high frequencies [17]. For this experiment, it is calculated that a 300 watt power amplifier will be enough to output appropriate voltage at maximum frequency (150 MHz) if the sample is 3mm by 3mm and if the sample is less than 1 micron thick. With this high of power comes a need for a proper cooling mechanism, otherwise the components will exceed their rated maximum temperature and become non-functional circuit elements. A cooling system will be implemented to where a pump continuously flows chilled oil through a plastic chamber, in which the circuit will sit. This will effectively cool the components down when the circuit is operational. The copper traces between components has been maximized, so as to increase surface area and allow great dissipation of heat.

The basic circuit design is shown in *Figure 10*. The basic schematic is similar to the low frequency system, except that 50Ω terminating resistors are now being used. An SMA board launch is utilized to supply signal to the board. The entire bottom side of the board will be a ground plane and vias will be channeled from the top to bottom layer where a ground is required in the top layout. As before, a surface mount capacitor (1 μF) is used as the reference capacitor. The resistors are surface mount terminating resistors, which are primarily used to sink most of the signal to ground and prevent reflections. These are ideal for this circuit since they have a very high power rating. The sample will be held in a capacitive geometry by a fuse-spring. The spring will physically push the sample onto a pad underneath the sample (this is not shown in the figure). Braided Litz wire will be soldered to the copper island that is between the sample and the reference capacitor, as well as the island between the resistors. These wires will come vertically off the board and out of the cooling chamber, where probes will be attached to measure the x- and y-channel voltages. It is important to understand the frequency and voltage limitations of each component and measurement device well before proceeding.

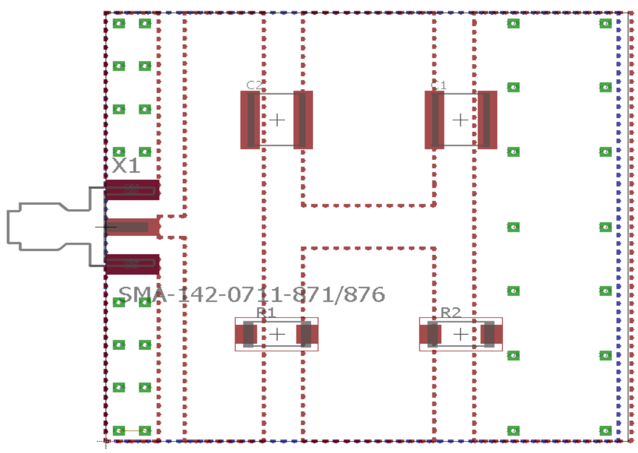


Figure 10: High Frequency Sawyer-Tower PCB layout.

Analysis

Once the test has been performed and the data has been obtained by the digital oscilloscope, it is important to streamline calculations and quickly extract useful P-E values. At this time, no modular program has been applied to this system, but this implementation would not be difficult and would greatly increase efficiency of measurements. If it were to be implemented, a software program, such as Python, could be used to immediately convert voltage values obtained by the digital oscilloscope to polarization or electric field values based on input parameters, such as sample area, thickness, reference capacitance, etc. A user would initially be prompted with input parameters, and this values would be entered. As was done earlier, calculations of P and E can be reduced to a constant multiplier times the respective voltage:

$$P = \beta * V_y$$

$$E = \gamma * V_x$$

where $\beta = \frac{C_2}{A}$ = constant and $\gamma = \frac{R_1 + R_2}{d * R_2}$ = constant

Input parameters would automatically calculate β and γ and the program would know to multiply x-channel voltages by β to obtain the electric field values. A similar procedure could obviously be done for the polarization. If one decided to change sample thickness or any other parameter, the user would simply be prompted to enter the new input parameters before testing and the P-E loops would be obtained without having to recalculate the conversion by hand. Once the P-E loop has been obtained, one can simply tell the software program to extract the values from this loop.

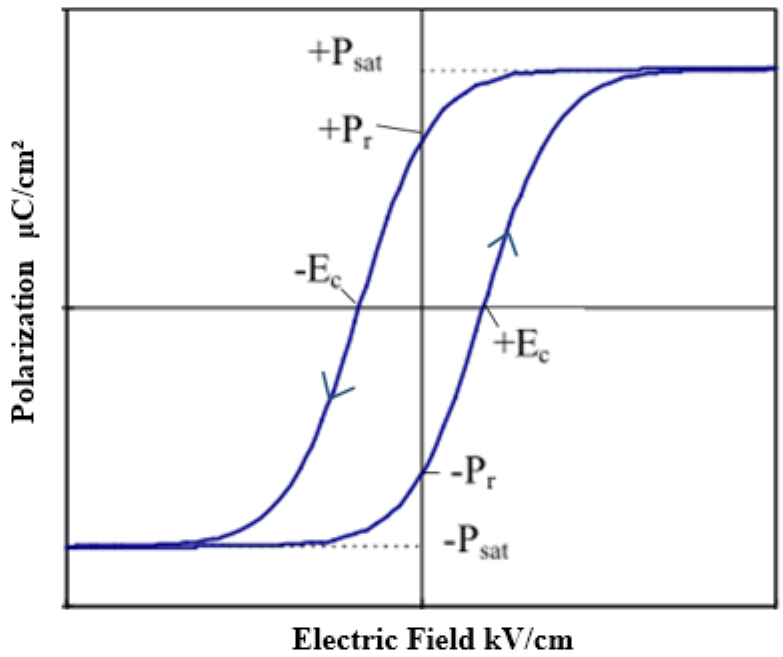
The software can be utilized to obtain the remnant polarization by finding the point at which the absolute value of electric field is at a minimum (the remnant polarization is the point at which the hysteresis curve crosses the y-axis). This will result in two numbers, one for the positive remnant polarization value and one for the negative remnant polarization value. Ideally, the absolute value of these two numbers would be equal, but a vertical shift of the hysteresis curve is possible. In this case, the absolute values of both positive and negative remnant polarization values must be added together and this value must be divided by two to obtain the true remnant polarization value:

$$P_r = \frac{|+P_r| + |-P_r|}{2}$$

The exact same mathematical treatment can be done for coercive electric field, which is the point at which the hysteresis curve crosses the x-axis.

$$E_c = \frac{|+E_c| + |-E_c|}{2}$$

Finding the saturation polarization is a little trickier. Since the saturation polarization is, strictly speaking, not the maximum value of polarization as was mentioned earlier in this essay, one must calculate the slope as the curve approaches the maximum and minimum E-field value and consequently trace that line back to the y-axis. Let's quickly take another look at the hysteresis curve, same as *Figure 3*.



With the hysteresis loop travelling in the counter-clockwise direction, the P and E values increase up until the maximum P values, the loop then abruptly begins travelling left to right. At this point, when the E values begin to decrease (at the top of the loop), the slope of the loop should be evaluated. This means taking a small selection of data points when E begins decrease and using software to calculate the slope with those data points. It is then fairly simple to mathematically "trace back" the line to the y-axis. This can be done by using the point-slope formula y = mx + b. The y-value, slope (m), and the x-value at which E is maximum are all known. The point b will simply be the point at which the line crosses the y-axis, which is exactly the saturation polarization (Psat).

$b = y - mx$

In terms of P-E values, this translates to:

$+P_{sat} = +P_{max} - (+E_{max})(slope)$

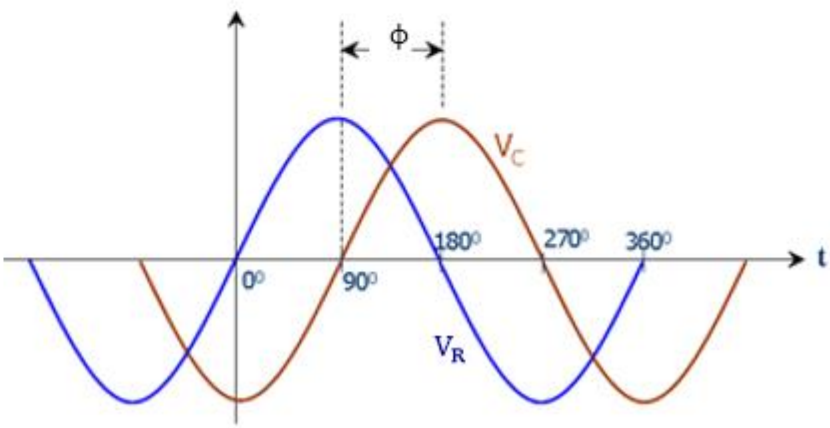
This will yield the saturation polarization value at the *top* part of the loop. The negative saturation polarization is calculated with the same procedure as before:

$-P_{sat} = -P_{max} - (-E_{max})(slope)$

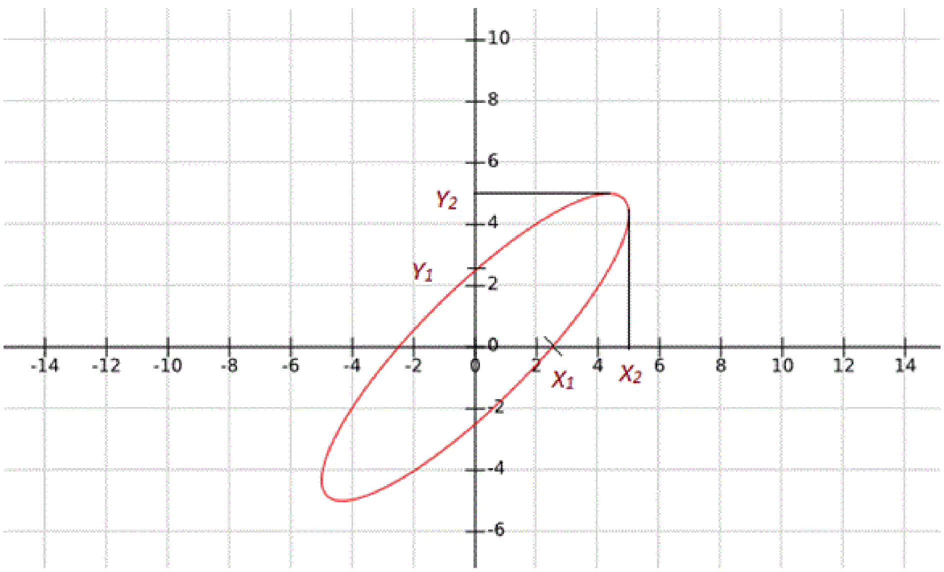
Note that the slope should be positive for both cases. For the negative saturation polarization (-Psat), one should realize that the slope is to be calculated when the E-values begin increasing (and hence the curve begins travelling left to right). As was discussed previously, it is possible for the curve to have a vertical shift. In this case, the true saturation polarization value will be:

$$P_{sat} = \frac{|+P_{sat}| + |-P_{sat}|}{2}$$

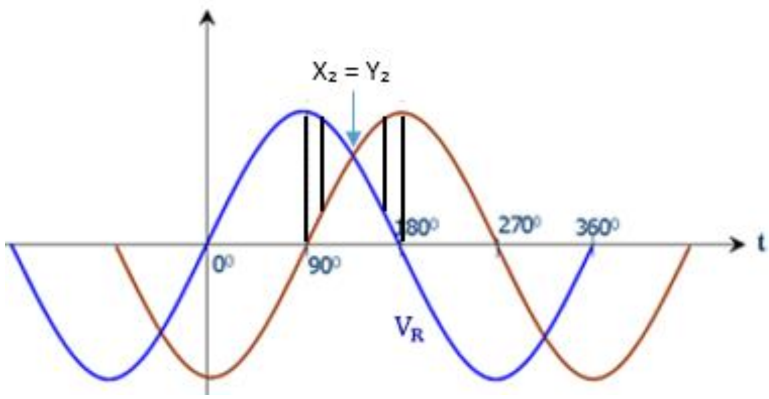
To calculate the area of the curve, it is often simpler to look at the raw waveforms of the hysteresis loop. This means that, instead of viewing the data in x-y mode, one can simply look at the x-channel and y-channel waveforms together. It is easier to analyze a simple x-y plot of a resistor and capacitor and analyze how the area of this loop can be calculated using the raw waveforms. Remember that there will be a phase shift when a capacitor and resistor are in parallel. This shift will look something like:



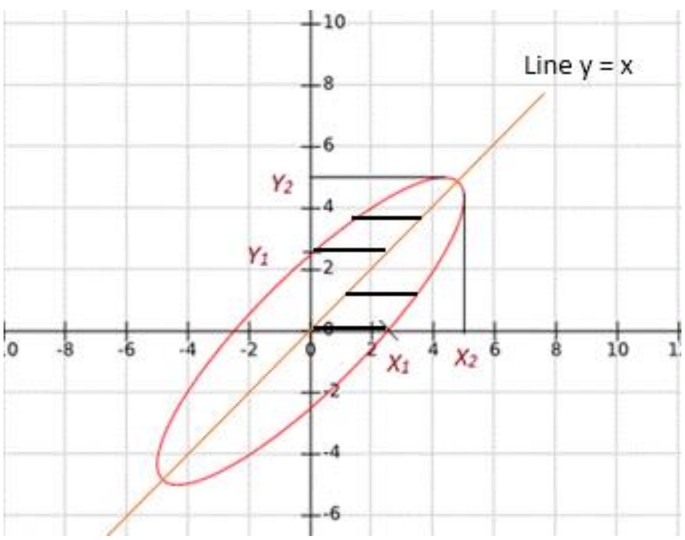
With the resulting x-y plot:



It is obvious that the phase shift and amplitude differences in the raw waveforms will result in openness in the x-y plot. It is easy to imagine two resistors in parallel with the known waveforms and x-y plots measured. The waveforms would be perfectly in phase and thus a diagonal line (y = x line) would appear. To calculate the area, one can subtract the data points in the x-channel waveform minus the data points in the waveform for the y-channel. Obviously, since the resistors waveforms are exactly equal, the subtraction will lead to zero at all times, resulting in no area as is expected for a diagonal line. If one algebraically sums all of these differences for a full cycle, one would obtain an approximate area for the loop. To demonstrate this, I will show what exactly subtracting the waveforms means in terms of the x-y plot. Look at the x-y plot for the capacitor/resistor network shown above, note that at point X_1, $Y = 0$. At point Y_2 and X_2, the values are equal. This means that values on the waveforms plots are equal. At point Y_1, $X = 0$.



The black bars represent the subtraction of the two waveforms. This coincides with the black bars on the x-y plot shown below.



Hopefully, this makes it easier to see how each subtracted data points in the raw waveforms will create a line on the x-y loop, and these lines can then be added up over a cycle to give loop area. As an aside, any continuous and periodic function produced by the signal generator will form a loop if x- and y-channels are plotted. This is because the periodic waveforms, at some point, must be equal to each other due to the mean-value theorem within a given period. Thus, a loop will always result. At any rate, the waveforms (consisting of data points at discrete times) can be evaluated in software, such as Python. The lines, when summed over an entire cycle, will approximate the area of the loop. Thus, it is possible to perform an integration with respect to time over a full cycle (period T) to give us loop area. This would work only if some function describing the x- and y-channel waveforms could be derived (or at least approximated) using software. Remember that ultimately the *P-E Loop Area* is desired. Thus, the conversion from voltages to polarization and electric field is required:

$$\int_0^T \gamma x(t) - \beta y(t)\, dt = \int_0^T E(t) - P(t) dt$$

where x(t) is the raw x-channel waveform, y(t) is the raw y-channel waveform, γ is the converting factor from voltage to electric field, β is the converting factor from voltage to polarization, and E(t) and P(t) are the converted waveforms as functions of time.

Lastly, one can calculate electric susceptibility and therefore relative permittivity as a function of electric field if one is presented with a P-E loop. The slope of the P-E loop is directly related to the capacitance and therefore the relative permittivity [4], [6]. Looking back at the P-E loop for the simple dielectric capacitance (*Figure 2a*), it is a straight line [6]. The slope of this line is constant, and thus susceptibility and relative permittivity will be constant. This makes sense, due to the linear nature of dielectrics in capacitive geometries. The P-E loop for ferroelectrics is non-linear, and a strict value for relative permittivity does not exist, as it changes with applied field. The relationship between polarization, electric field, and susceptibility is:

$$\vec{P} = \epsilon_0 X_e \vec{E}$$

where E is the electric field vector, ε_o is the permittivity of free space, X_e is the electric susceptibility, and P is the polarization vector. As was mentioned, the slope of the P-E loop is intimately related to the electric susceptibility. Taking the derivative of P with respect to E will give us the slop of the loop:

$$\frac{dP}{dE} = \epsilon_0 X_e$$

$$X_e = \left(\frac{1}{\epsilon_0}\right)\left(\frac{dP}{dE}\right)$$

Remember that relative permittivity is (ϵ_r) is related to electric susceptibility with this equation:

$$\epsilon_r = 1 + X_e$$

Thus, relative permittivity can be solved from the slope of the P-E loop using this relation:

$$\epsilon_r(E) = 1 + \left(\frac{1}{\epsilon_0}\right)\left(\frac{dP}{dE}\right)$$

The graph of the relative permittivity will look something like *Figure 11*.

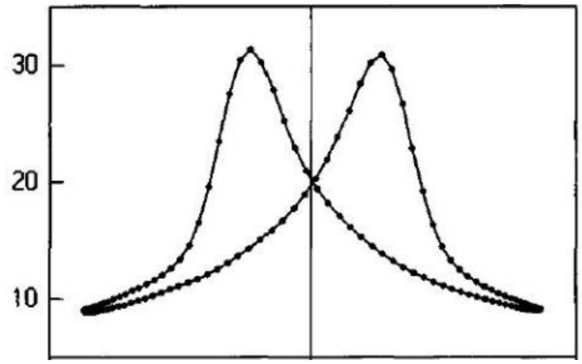


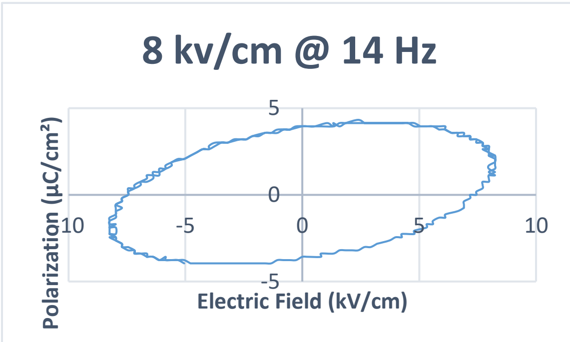
Figure 11: Relative permittivity plotted against electric field. Note that this is reflective across the y-axis. This symmetry is due to the loop symmetry, and for each positive slop that is calculated from the loop running left to right, one would expect the same negative slope when the loop is running right to left.

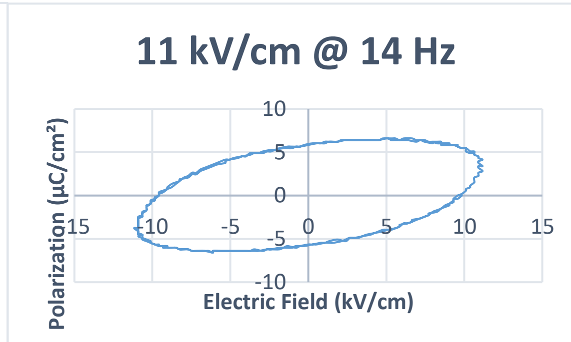
Combining all of these, one can calculate the: remnant polarization, coercive electric field, saturation polarization, hysteresis area, and relative permittivity given a P-E loop and data points.

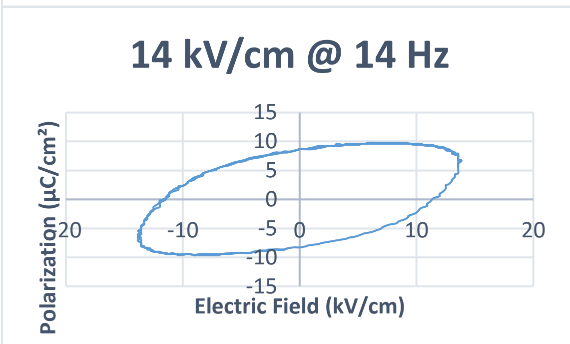
Results

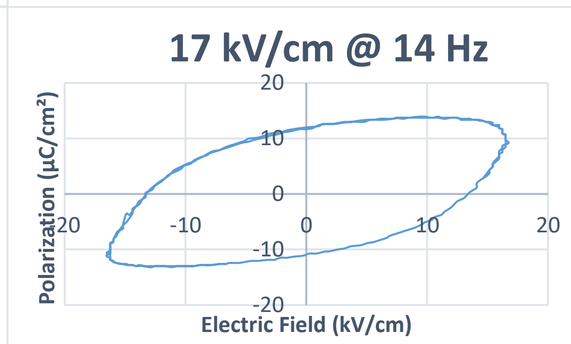
Hysteresis loops for the low frequency Sawyer-Tower circuit have been obtained at various field strengths. A Siglent SDG1050 Signal Generator produced a sine wave at a frequency of 14 Hz with voltage outputs increasing from 150 mV to 500 mV in increments of 50 mV. This produced maximum output voltage (after signal amplification) varying from 250 V to 840 V and maximum electric field strengths varying from 8 kV/cm to 28 kV/cm. A separate oscilloscope with two regular test probes were attached to the amplifier (and ground reference) to ensure that the waveform leaving the amplifier was a clean signal. This was done because sufficiently high voltage output from the signal generator at sufficiently low frequency can occasionally cause signal distortion. Two resistors, both 10 kΩ, were placed in series. This was wired in parallel to a PZT-5H sample with area 42.5 mm^2 and thickness 150 microns and a reference capacitor (capacitance 1 μF) in series. The PZT sample was covered in EPO-TEK OE132-43 Epoxy to prevent arcing. Two Tektronix P5100A high voltage probes were used to extract input signal and signal between capacitors. An OWON SDS7072V digital oscilloscope was used to readout voltages. The oscilloscope's probe attenuation settings and voltage scaling was adjusted prior to testing to ensure correct readout. The oscilloscope was connected to a computer via USB to digitally record the waveforms on the OWON oscilloscope software.

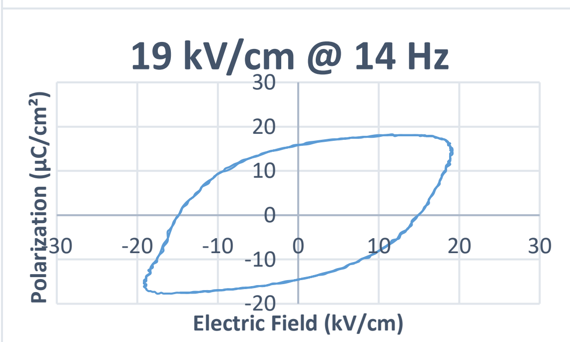
The output voltage on the signal generator was initially set to 50 mV and the waveform was recorded on the oscilloscope software. This same procedure occurred in increments of 50 mV up to 500 mV, resulting in 8 data sets. The data sets that were saved on the oscilloscope software could be easily transferred to an excel spreadsheet, where the P-E loop was created and polarization characterization measurements took place. These measurements followed exactly from how they were calculated in the section above, using built in excel functions. Voltages were converted to polarization or electric field by the applying the appropriate conversion factors in separate columns and those values were graphed. The graphs, as well as the polarization measurements, are shown below.

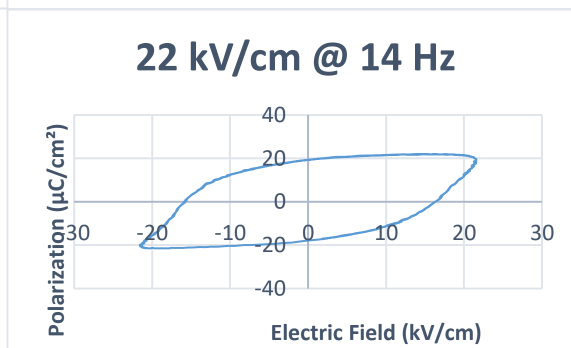


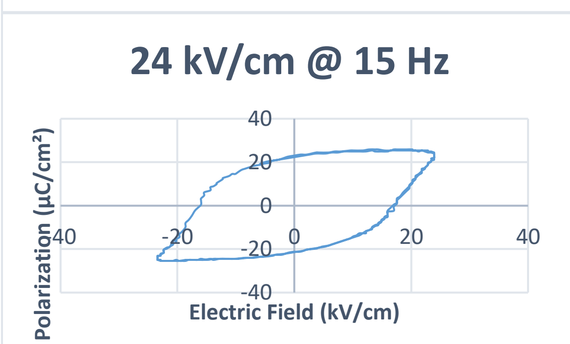


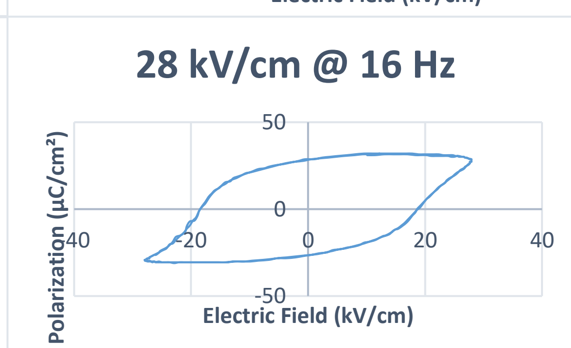












Field Strength (kV/cm)	Area (mC/cm^3)	Remnant Polarization ($\mu C/cm^2$)	Coercive Electric Field (kV/cm)	Saturation Polarization ($\mu C/cm^2$)	Maximum Electric Field (kV/cm)
8 kV/cm	709.7195	3.57656	7.448	4.32952	8.246
11 kV/cm	945.0558	5.83544	9.576	6.5884	11.172
14 kV/cm	1107.302	8.65904	11.438	9.78848	13.832
17 kV/cm	1238.207	12.04736	13.3	13.92976	16.758
19 kV/cm	1744.01	14.68272	14.896	18.25928	19.152
22 kV/cm	1756.55	19.38872	15.694	22.21232	21.546
24 kV/cm	1884.382	21.177	17.024	25.883	23.94
28 kV/cm	2287.866	28.7066	18.62	32.0008	27.93

Note that the 24 kV/cm and 28 kV/cm data sets were set at 15 Hz and 16 Hz respectively, instead of 14 Hz. This is because slight signal distortion began to occur in the audio amplifier. It is obvious that the hysteresis loop had not yet reached saturation. Higher voltage should have been applied, but a malfunction in the amplifier prevented further testing. At any rate, these values match quite well with the values that other studies have achieved, helping to validate the Sawyer-Tower approach at low frequencies. The calculation was done in Microsoft Excel using the techniques mentioned in the previous section. Calculations of relative permittivity were excluded as the derivative function (to find slope between data points) was not available.

Testing has not been done on the medium systems yet, but the design that was mentioned earlier in this essay is currently being produced. The high frequency system has also not been tested, as the PCB board has not been manufactured. Preliminary testing will be done on the high frequency system (using the same surface mount capacitors and resistors that will be used in high frequency testing) but will instead be tested at medium frequencies to assure the circuit is working.

Conclusion

Ultimately, the Sawyer-Tower approach has been successful in demonstrating P-E loops, in our case, for PZT-5H. The values that were achieved, and the overall shape of the loop at high field strengths show close similarities to other studies of this nature done on PZT-5H. This, in itself, helps verify the calculation techniques that were mathematically hypothesized in the previous section. The cost of each system was relatively low, compared to the alternative of purchasing a commercial system. As a matter of fact, almost all of the equipment required for the low frequency system was salvaged or borrowed from a physics lab. The only purchase that was necessary was the high voltage resistors and capacitors, which were inexpensive. Higher frequency testing is soon to be done and will be reported in another essay. The only theoretical limitations that would hinder this circuit from working would be stray inductances and capacitance caused by high frequencies, as well as possible reflection in the circuit. The design that was described above was made in hopes of mitigating these limitations, hopefully allowing the Sawyer-Tower method to analyze ferroelectrics into the MHz frequency range.

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